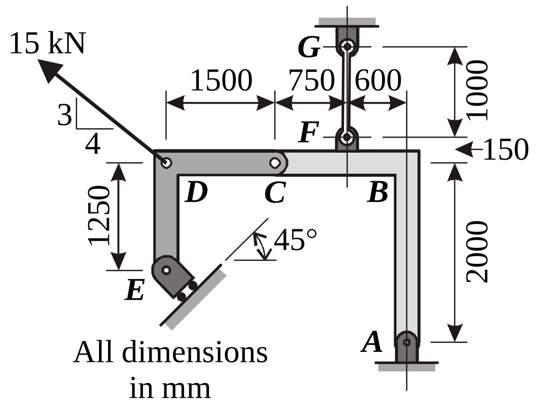
Question 1:



Given:

$L_{AB} = 2000 \cdot mm$ $\quad L_{BC} = 1350 \cdot mm$

$L_{CD} = 1500 \cdot mm$ $\quad L_{DE} = 1250 \cdot mm$

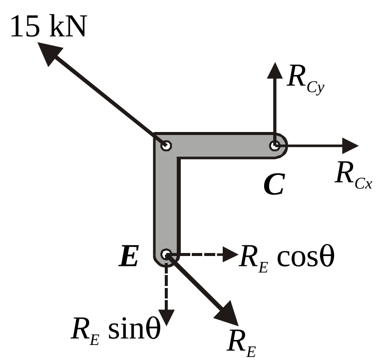
$L_{BF} = 600 \cdot mm$ $\quad \theta_E = 45 \cdot deg$

$F_D = 15 \cdot kN$

$d_C = 16 \cdot mm$ $\quad \tau_u = 80 \cdot MPa$

$\sigma_{all} = 175 \cdot MPa$

FBD of Member CDE:



$\Sigma M_C = 0$

$$-\left(\frac{3}{5} \cdot F_D\right) \cdot (L_{CD}) + (R_E \cdot \sin(\theta_E)) \cdot (L_{CD}) \ldots = 0$$
$$+ (R_E \cdot \cos(\theta_E)) \cdot (L_{DE})$$

$$R_E = \frac{3}{5} \cdot F_D \cdot \frac{L_{CD}}{\sin(\theta_E) \cdot L_{CD} + \cos(\theta_E) \cdot L_{DE}}$$

$R_E = 6.943\,kN$ (acting in direction shown)

$\Sigma F_x = 0$ $\quad -\left(\frac{4}{5} \cdot F_D\right) + R_{Cx} + R_E \cdot \cos(\theta_E) = 0$

$R_{Cx} = \frac{4}{5} \cdot F_D - R_E \cdot \cos(\theta_E)$ $\quad R_{Cx} = 7.091\,kN$ (acting to right)

$\Sigma F_x = 0$ $\quad \left(\frac{3}{5} \cdot F_D\right) + R_{Cy} - R_E \cdot \sin(\theta_E) = 0$

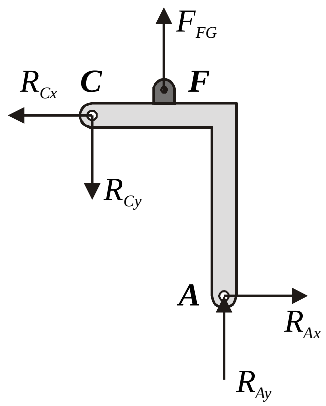
$R_{Cy} = \frac{-3}{5} \cdot F_D + R_E \cdot \sin(\theta_E)$ $\quad R_{Cy} = -4.091\,kN$ (acting downward)

(a) Factor of safety in Pin at Point C

- Resultant shear force in pin (single shear): $V_C = \sqrt{R_{Cx}^{\,2} + R_{Cy}^{\,2}}$ $\quad V_C = 8.186\,kN$
- Cross-sectional area: $A_C = \frac{\pi}{4} \cdot d_C^{\,2}$ $\quad A_C = 201.062 \times 10^{-6}\ m^2$
- Ultimate shear strength: $V_u = \tau_u \cdot A_C$ $\quad V_u = 16.085\,kN$
- Factor of safety: $FOS = \frac{V_u}{V_C}$ $\quad FOS = 1.965$

(b) Required size of Member FG

FBD of Member ABC:



$\Sigma M_A = 0$

$$(R_{Cx})\cdot(L_{AB}) + (R_{Cy})\cdot(L_{BC}) - (F_{FG})\cdot(L_{BF}) = 0$$

$$F_{FG} = \frac{R_{Cx}\cdot L_{AB} + R_{Cy}\cdot L_{BC}}{L_{BF}} \qquad F_{FG} = 14.432\,\text{kN}$$

(in tension)

$\Sigma F_x = 0 \qquad -R_{Cx} + R_{Ax} = 0 \qquad R_{Ax} = R_{Cx}$

$R_{Ax} = 7.091\,\text{kN}$

$\Sigma F_y = 0 \qquad -R_{Cy} + F_{FG} + R_{Ay} = 0$

$R_{Ay} = R_{Cy} - F_{FG} \qquad R_{Ay} = -18.523\,\text{kN}$

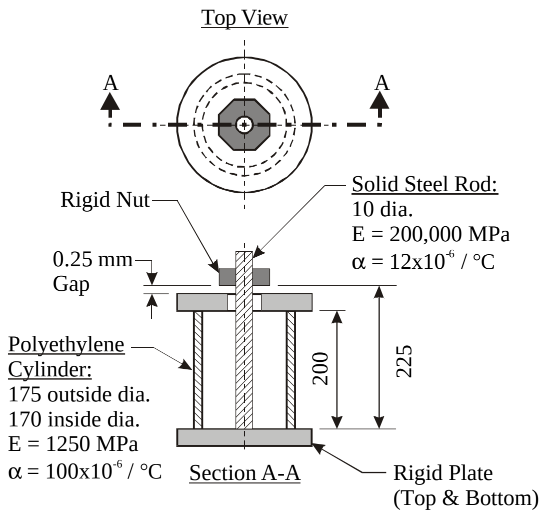
(acting downward)

Required size of Member FG:

$$A_{req} = \frac{|F_{FG}|}{\sigma_{all}} \qquad A_{req} = 82.468\times10^{-6}\ \text{m}^2$$

$$b = \sqrt{A_{req}} \qquad b = 9.081\ \text{mm}$$

$MPa := 10^6 \cdot Pa$ $kN := 10^3 \cdot N$



Input data: $\Delta T := 80 \cdot deg$

$\delta_{gap} := 0.25 \cdot mm$

- Cylinder: $L_c := 200 \cdot mm$

$d_o := 175 \cdot mm$ $d_i := 170 \cdot mm$

$E_c := 1250 \cdot MPa$

$$\alpha_c := \frac{100 \cdot 10^{-6}}{deg}$$

- Rod: $L_r := 225 \cdot mm$

$d_r := 10 \cdot mm$

$E_r := 200000 \cdot MPa$

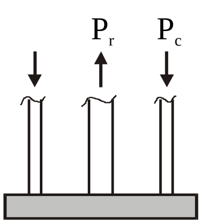
$$\alpha_r := \frac{12 \cdot 10^{-6}}{deg}$$

Thermal expansion:

$\delta_{c\_T} := \alpha_c \cdot \Delta T \cdot L_c$ $\delta_{c\_T} = 1.6\ mm$

$\delta_{r\_T} := \alpha_r \cdot \Delta T \cdot L_r$ $\delta_{r\_T} = 0.22\ mm$

Equilibrium:



$\Sigma F_y = 0$ $P_r - P_c = 0$

$P_r = P_c$

Cross Sectional Areas:

$$A_c := \frac{\pi}{4} \cdot \left(d_o^2 - d_i^2\right) \qquad A_c = 1354.81\ mm^2$$

$$A_r := \frac{\pi}{4} \cdot d_r^2 \qquad A_r = 78.54\ mm^2$$

Axial deformation:

$$\delta_{c\_P} = \frac{(-P_c) \cdot L_c}{A_c \cdot E_c} = \frac{(-P_r) \cdot L_c}{A_c \cdot E_c} \qquad \frac{L_c}{A_c \cdot E_c} = 118.1 \times 10^{-9} \frac{m}{N}$$

$$\delta_{r\_P} = \frac{P_r \cdot L_r}{A_r \cdot E_r} \qquad \frac{L_r}{A_r \cdot E_r} = 14.32 \times 10^{-9} \frac{m}{N}$$

Geometric compatibility:

$$\delta_{c\_P} + \delta_{c\_T} - \delta_{gap} = \delta_{r\_P} + \delta_{r\_T}$$

$$\frac{(-P_r)\cdot L_c}{A_c\cdot E_c} + \delta_{c\_T} - \delta_{gap} = \frac{P_r\cdot L_r}{A_r\cdot E_r} + \delta_{r\_T}$$

$$P_r := \frac{-\delta_{r\_T} + \delta_{c\_T} - \delta_{gap}}{L_r\cdot A_c\cdot E_c + L_c\cdot A_r\cdot E_r}\cdot A_r\cdot E_r\cdot A_c\cdot E_c \qquad P_r = 8.564\,\mathrm{kN} \qquad \sigma_r := \frac{P_r}{A_r} \qquad \sigma_r = 109.03\,\mathrm{MPa}$$

$$P_c := P_r \qquad \sigma_c := \frac{-P_c}{A_c} \qquad \sigma_c = -6.32\,\mathrm{MPa}$$

Deformations and final lengths:

$$\delta_{c\_P} := \frac{(-P_c)\cdot L_c}{A_c\cdot E_c} \qquad \delta_{c\_P} = -1.011\,\mathrm{mm}$$

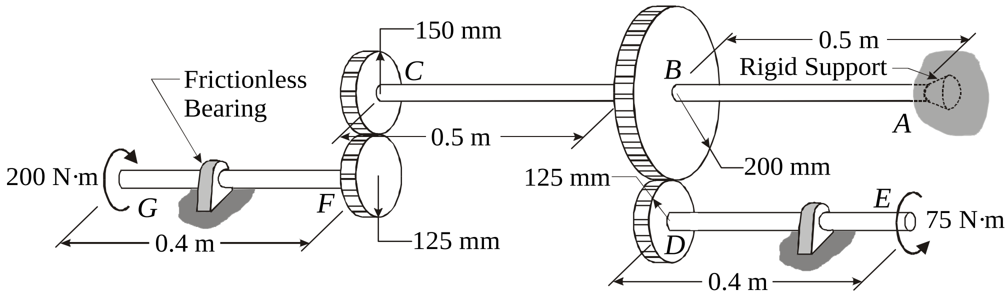
$$L_{c\_final} := L_c + \delta_{c\_P} + \delta_{c\_T} \qquad L_{c\_final} = 200.589\,\mathrm{mm}$$

$$\delta_{r\_P} := \frac{(P_r)\cdot L_r}{A_r\cdot E_r} \qquad \delta_{r\_P} = 0.123\,\mathrm{mm}$$

$$L_{r\_final} := L_r + \delta_{r\_P} + \delta_{r\_T} \qquad L_{r\_final} = 225.339\,\mathrm{mm}$$

$$\left(\delta_{c\_P} + \delta_{c\_T}\right) - \left(\delta_{r\_P} + \delta_{r\_T}\right) = 0.25\,\mathrm{mm}$$

Question 3:



Given: $d = 40 \cdot mm$

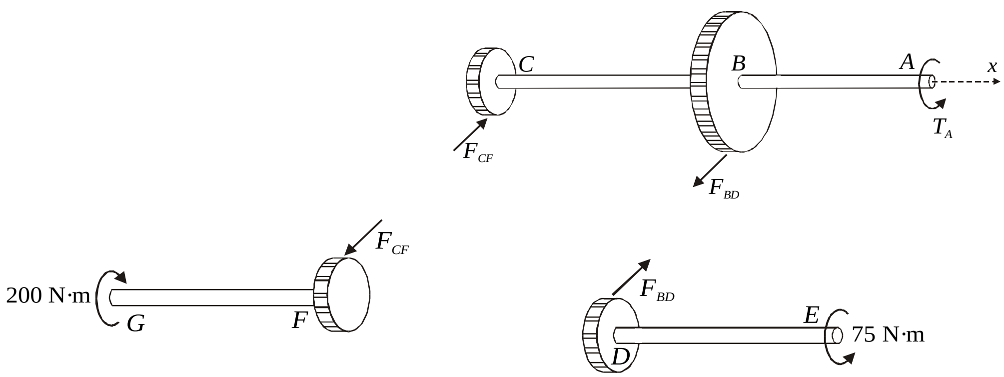
$L_{AB} = 500 \cdot mm$ $L_{BC} = 500 \cdot mm$ $L_{DE} = 400 \cdot mm$ $L_{FG} = 400 \cdot mm$

$c_B = 200 \cdot mm$ $c_C = 150 \cdot mm$ $c_D = 125 \cdot mm$ $c_F = 125 \cdot mm$

$T_E = 75 \cdot N \cdot m$ $T_G = 200 \cdot N \cdot m$ $G = 77000 \cdot MPa$

Solution:

FBD's: (Neglecting all reactions that don't produce moments about x-axis)



Shaft DE:

$\Sigma M_x = 0$ $T_E - (F_{BD} \cdot c_D) = 0$ $F_{BD} = \dfrac{T_E}{c_D}$ $F_{BD} = 600\ N$ in direction shown

Shaft FG:

$\Sigma M_x = 0$ $-T_G + (F_{CF} \cdot c_F) = 0$ $F_{CF} = \dfrac{T_G}{c_F}$ $F_{CF} = 1600\ N$ in direction shown

Shaft ABC:

$\Sigma M_x = 0$ $(F_{CF} \cdot c_C) - (F_{BD} \cdot c_B) + T_A = 0$ $T_A = -F_{CF} \cdot c_C + F_{BD} \cdot c_B$

$T_A = -120 N \cdot m$

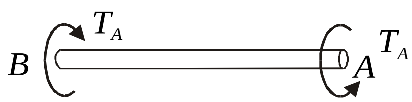
<u>Part (a):</u> Shear stress in shaft AB $\quad d = 40\,\text{mm}$

- Polar moment of inertia (all shafts): $J = \frac{\pi}{2}\cdot\left(\frac{d}{2}\right)^4 \qquad J = 251.33 \times 10^{-9}\ \text{m}^4$

- Shear stress: $\tau_{AB} = \dfrac{|T_A|\cdot\left(\frac{d}{2}\right)}{J} \qquad \tau_{AB} = 9.55\,\text{MPa}$

<u>Part (b):</u> Angle of twist at Point E $\quad \phi_A = 0\cdot\text{rad}$ (rigid support)

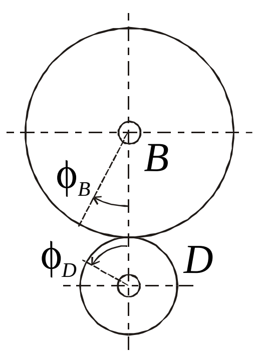
- Shaft AB:



$$\phi_{BA} = \frac{-T_A\cdot L_{AB}}{J\cdot G} \qquad \phi_{BA} = 3.1 \times 10^{-3}\,\text{rad}$$

$$\phi_B = \phi_{BA} + \phi_A \qquad \phi_B = 3.1 \times 10^{-3}\,\text{rad}$$

- Angle of twist of gear D: Same arc length travelled by both gears

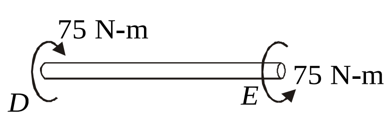


Looking from right to left

$$\phi_B\cdot c_B = -\phi_D\cdot c_D$$

$$\phi_D = -\phi_B\cdot\frac{c_B}{c_D} \qquad \phi_D = -4.96 \times 10^{-3}\,\text{rad}$$

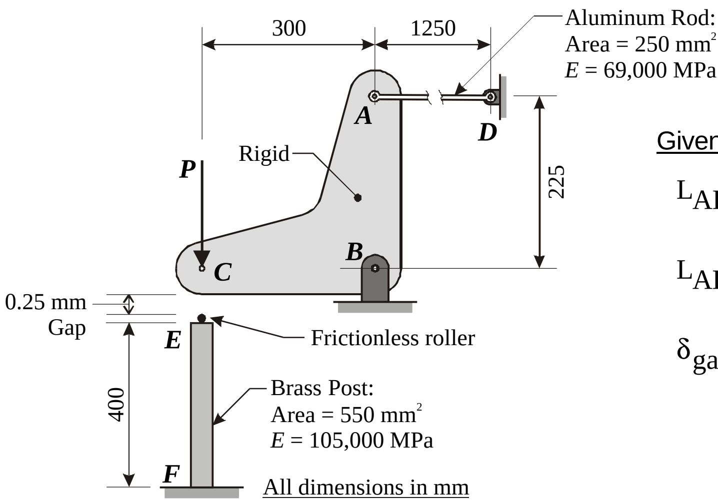
- Shaft DE:



$$\phi_{ED} = \frac{T_E\cdot L_{DE}}{J\cdot G} \qquad \phi_{ED} = 1.55 \times 10^{-3}\,\text{rad}$$

$$\phi_E = \phi_{ED} + \phi_D \qquad \phi_E = -3.41 \times 10^{-3}\,\text{rad}$$

Question 4:



Given:

$L_{AB} = 225 \cdot mm$ $L_{BC} = 300 \cdot mm$

$L_{AD} = 1250 \cdot mm$ $L_{EF} = 400 \cdot mm$

$\delta_{gap} = 0.25 \cdot mm$

- Aluminum rod: $A_{al} = 250 \cdot mm^2$ $E_{al} = 69000 \cdot MPa$

- Brass post: $A_{br} = 550 \cdot mm^2$ $E_{br} = 105000 \cdot MPa$ $\varepsilon_{br} = -0.0014$

Solution:

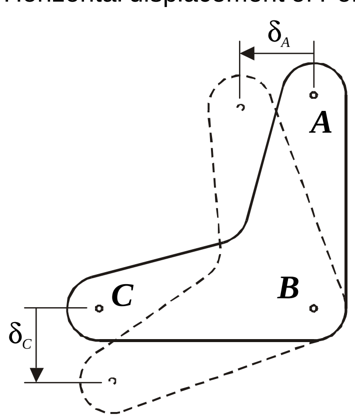
- Axial deformation of the brass post: $\delta_{EF} = \varepsilon_{br} \cdot L_{EF}$ $\delta_{EF} = -560 \times 10^{-6}\ m$

Since $\delta_F = 0 \cdot m$ $\delta_E = \delta_{EF} + \delta_F$ $\delta_E = -560 \times 10^{-6}\ m$

- Total vertical displacement of Point C:

$\delta_C = \delta_E - \delta_{gap}$ $\delta_C = -810 \times 10^{-6}\ m$

- Horizontal displacement of Point A: (small displacement thoery)



Similar triangles:

$$\frac{\delta_A}{L_{AB}} = \frac{-\delta_C}{L_{BC}}$$

$$\delta_A = \frac{-\delta_C}{L_{BC}} \cdot L_{AB} \qquad \delta_A = 607.5 \times 10^{-6}\ m$$

(assume +ve to left)

- Axial deformation of aluminum rod: $\delta_D = 0 \cdot m$

$$\delta_{AD} = \delta_A - \delta_D \qquad \delta_{AD} = 607.5 \times 10^{-6}\ m$$

- Axial forces:
  - Brass post (compression):

$$\delta_{EF} = \frac{(-F_{EF}) \cdot L_{EF}}{A_{br} \cdot E_{br}} \qquad F_{EF} = \frac{-\delta_{EF}}{L_{EF}} \cdot A_{br} \cdot E_{br}$$

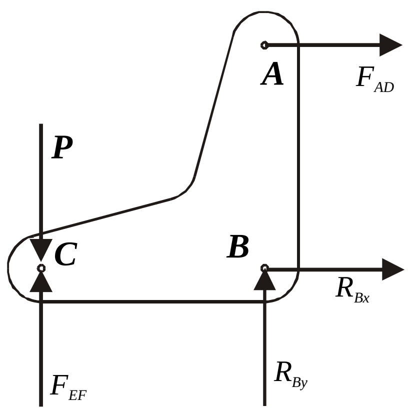
$$F_{EF} = 80.85 \times 10^3 \text{ N}$$

  - Aluminum rod (tension)

$$\delta_A = \frac{F_{AD} \cdot L_{AD}}{A_{al} \cdot E_{al}} \qquad F_{AD} = \frac{\delta_A}{L_{AD}} \cdot A_{al} \cdot E_{al}$$

$$F_{AD} = 8.383 \times 10^3 \text{ N}$$

- FBD of bracket ABC:



$$\Sigma M_B = 0 \quad \text{+ve}$$

$$(-F_{AD}) \cdot (L_{AB}) - (F_{EF}) \cdot (L_{BC}) + P \cdot (L_{BC}) = 0$$

$$P = \frac{F_{AD} \cdot L_{AB} + F_{EF} \cdot L_{BC}}{L_{BC}}$$

$$P = 87.138 \times 10^3 \text{ N}$$ (in direction shown)